ICAR '97
Monterey, CA, July 7-9, 1997

Dynamic Force/Torque Sensors: Theory and Experiments*

A. Bicchi A. Caiti D. Prattichizzo

Dipartimento di Sistemi Elettrici ed Automazione, Università di Pisa
via Diotisalvi, 2, 56100 Pisa, Italia.
bicchi(caiti)[prattichizzo]@dsea.unipi.it

Abstract

Although present–day Force/Torque sensors are mostly designed and used as if they were quasi–static devices, if significant compliance and/or stringent requirements on measurements bandwidth are in order, a dynamic analysis of such sensors is necessary. In this paper we consider the optimal (in a worst–case sense) design of F/T sensors, based on distributed–parameter models of its compliance, and algorithms that can be used to obtain F/T measurements in real time with high bandwidth. Theoretical expectations are confirmed by experimental results.

1 Introduction

In many situations arising in robotic systems, one is interested in measuring the forces and torques applied at the terminal point of a certain mechanical structure. This information is usually obtained from strain gauges conveniently placed on the structure itself. The estimation of the applied force (or torque) from the strain measurements is most often obtained by assuming a quasi-static relation between the force applied and the strain. This assumption is usually well respected even if the force is time varying, provided that the mechanical structure is rigid enough to make negligible the effects of vibration in the structure itself. This is the case, for instance, of force/torque sensors mounted at the end-effector of robot manipulators.

Whenever the flexibility of the mechanical structure can not be neglected, the measured strain will include components due to the vibration of the structure, and the estimation of the applied, time-varying, force cannot be done by using quasi-static relations. To overcome this problem, the authors have recently proposed an approach to *dynamic* force/torque sensing that attempts to recover the applied force from the strain measurement by fully exploiting the dynamic nature of the force-strain relation (Bicchi *et al.*, 1997).

The estimation of the applied force, both in quasi-static and in full dynamic case, is dependent on the placement of the sensing device (the strain gauges) on the mechanical structure. In this paper some criteria for the selection of the optimal sensor placement are proposed, motivated from general consideration on multivariate sensor design (Bicchi and Canepa, 1994). Moreover, it is shown, by means of experimental results, how the algorithm introduced in (Bicchi *et al.*, 1997) can produce effective estimates of the applied force when the quasi-static approximation is not respected.

The analysis carried out in the rest of the paper specializes to the case of a flexible beam, and to the study of transversal vibrations . As such, it has to be considered as a preliminary investigation in order to proceed in the future towards more complex situations. However, even in this simple case, it appears the interesting result that the sensor design has to trade-off between the best accuracy of the solution and the stability of the inversion process.

The paper is organized as follows: in the next section, by using standard tools from truncated modal analysis, a system of ordinary differential equations is obtained for the case of a flexible beam. In Section 3, the invertibility properties of the system are investigated, a criterion for the design of the force sensor is proposed, and constraints on the design are discussed. In Section 4, a robust numerical algorithm for system inversion is described. Simulative and experimental results to validate the theoretical expectations are reported in Sections 5 and 6. Finally, conclusions and future perspectives are given.

*Authors wish to thank students M. Biancalani, F. Bonsignori, M. Lombardo and G. Zanobin for their work on the experimental device. This work has been partially supported by MURST.

0-7803-4160-0-7/97 $10.00 © 1997 IEEE

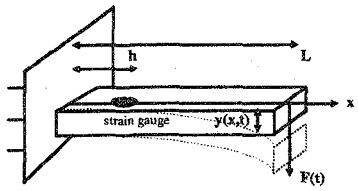

Figure 1: The physical system considered. A beam of length L, fixed at one end and free at the other end, is subject to a time-varying force $F(t)$ applied at the free end which causes transverse vibrations $y(x,t)$. A strain gauge, placed at a position h along the beam, is used to estimate the applied force.

2 Modal Analysis

Consider the transverse vibrations $y(x,t)$ excited in a flexible beam of length L fixed at one extremity by a time-varying point force $F(t)$ applied at the free end point. The mechanical system is schematically depicted in fig. 1. The beam is assumed isotropic, and of total mass m homogeneously distributed. A strain gauge is placed at a distance h from the fixed end point of the beam.

The aim of the paper is to design an algorithm able to retrieve the force $F(t)$ at any instant t from the measurement $s(h,t)$ of the strain gauge placed in the position h. The position of the gauge h is the free parameter of the design.

The monodimensional beam subject to transverse vibration is a well studied system (Meirovitch, 1967) governed by the Euler-Bernoulli equations:

$$EI\frac{\partial^4 y}{\partial x^4} + 2\xi\frac{\partial^4}{\partial x^4}\frac{\partial}{\partial t}y + m\frac{\partial^2 y}{\partial t^2} = F(t)\delta(x-L) \qquad (1)$$

where EI, ξ and m are the stiffness, structural damping and mass of the beam.

The solution of equation (1) can be expressed in terms of its normal modes decomposition as:

$$y(x,t) = \sum_{k=1}^{\infty} q_k(t)Y_k(x) \qquad (2)$$

where the terms $q_k(t)$ play the role of a weight in time and the normal modes $Y_k(x)$, $x \in [0,L]$ are defined as

$$\begin{aligned}&Y_k(x) = \\ &(sin(\beta_k L) - sinh(\beta_k L))(sin(\beta_k x) - sinh(\beta_k x)) + \\ &+(cos(\beta_k L) + cosh(\beta_k L))(cos(\beta_k x) - cosh(\beta_k x))\end{aligned} \qquad (3)$$

being β_k $(k = 1, \cdots, \infty)$ a solution of equation $cos(\beta L)cosh(\beta L) = 1$.

By truncating the modal expansion to the N-th mode, and by defining time weights and their derivative as the state vector:

$$\mathbf{x} = [q_1(t), \cdots, q_N(t), \dot{q}_1(t), \cdots, \dot{q}_N(t)]^T, \qquad (4)$$

the following system of ordinary differential equations is obtained from equation (1):

$$\begin{aligned}\dot{\mathbf{x}} &= A\mathbf{x} + Bu \\ y &= C\mathbf{x}\end{aligned} \qquad (5)$$

where $u(t) = F(t)$ is the input force to be estimated, $y = s(t)$ is the measurement signal from the strain gauge positioned at h, and matrices A, B and C assume the following structures

$$A = \begin{bmatrix} 0 & I_N \\ -(\frac{EI}{m})\Lambda^2 & -2\frac{\xi}{m}\Lambda \end{bmatrix} \qquad (6)$$

$$B = [0 \cdots 0 \; Y_1(L) \cdots Y_N(L)]^T \qquad (7)$$

$$C = [\frac{\partial^2 Y_1(h)}{\partial x^2} \cdots \frac{\partial^2 Y_N(h)}{\partial x^2} \; 0 \cdots 0], \qquad (8)$$

being matrix Λ a $N \times N$ diagonal matrix whose $k-th$ diagonal term is β_k.

3 Dynamic Inversion and Optimal Sensor Design

Consider the dynamic system in (5), where matrices A, B, and C have been previously defined for a fixed number N of modes , so that $A \in \Re^{2N\times 2N}$, $B \in \Re^{2N\times 1}$, $C \in \Re^{1\times 2N}$. The estimation of the input signal u, given the measurement y and knowing the system structure can be cast into a problem of *system inversion*. A necessary and sufficient condition for a system to be invertible has been given in (Brockett and Mesarovic, 1965). To our case this condition specializes in the following proposition.

Proposition 1 *Consider the matrix $M \in \Re^{4N-1\times 2N}$*

$$M = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{2N-1}B & \cdots & \cdots & CB \\ CA^{2N}B & \cdots & \cdots & CAB \\ \vdots & \ddots & \ddots & \vdots \\ CA^{4N-1}B & \cdots & \cdots & CA^{2N-1}B \end{bmatrix}. \qquad (9)$$

A necessary and sufficient condition for the invertibility of system (5) is that the rank of matrix M is $2N$.

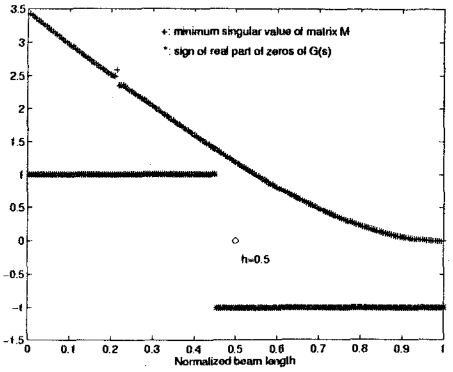

Figure 2: Cost function and constraints for the optimal design problem, computed with a 3-modes model. +: minimum singular value of M. *: sign of the maximum real part of the zeros of $G(s)$. o: suggested sensor placement accordingly to the defined criterion

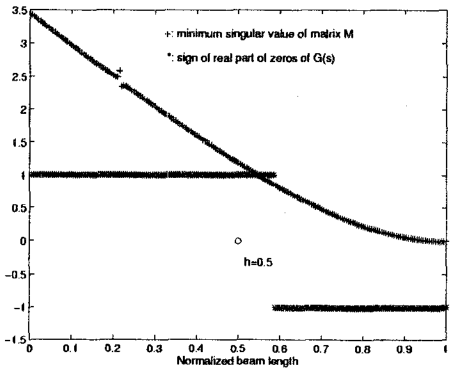

Figure 3: Cost function and constraints for the optimal design problem computed with a 4-modes model. Symbols as in fig. 2.

The rank test on matrix M is *not* the most computationally efficient test for system invertibility. Successively other methods to analyze the invertibility have been proposed in literature, as for instance those in (Sain and Massey, 1969), (Silverman, 1969), (Moylan, 1977), (Tan and Vandewalle, 1988), to name a few. However, for the aim of this paper the test on matrix M has been chosen since it plays the role of the "measurement matrix" for dynamic system in equation (6) (Bicchi and Canepa, 1994). For further details on this point the reader is referred to the proof of the above proposition as given in (Sain and Massey, 1969). It ensues that the knowledge of matrix M is relevant for the analysis of the system invertibility as pointed out in the following remark.

Remark 1 Matrix M gives information on the invertibility of the system, and on the properties of the inversion result, independently from the numerical algorithm that will be used.

The aim of this paper is to suggest an optimal criterion for the design of dynamic force sensors. The optimality is here meant as the maximization of the input-force/output-strain inversion accuracy. According to (Bicchi *et al.*, 1997), in this paper the maximization of the minimum singular values of M is proposed as the optimality criterion. Recall that matrix M depends, through the output matrix C, upon the gauge position h which represents the free parameter of the dynamic sensor design.

However, the inversion accuracy is not the unique requirement of the sensor design. In fact, as regards the inversion algorithm, a stability requirement on the inversion algorithm must be added. In terms of transfer functions (since a single input single output case is considered) this is equivalent to require that, for the resulting h, the transfer function of system (5) must *not* have any zero with positive real part. Being $G(s) = C(sI - A)^{-1}B$ the transfer function of (5), the design criterion takes the form:

$$\begin{array}{l} h^* = \arg\max_{h\in[0,L]} \sigma_{min}(M(h)) \\ \text{subject to: } Re(z_i(G(s))) < 0 \ \forall \ z_i(G(s)) \end{array} \tag{10}$$

where σ_{min} is the minimum singular value and $z_i(G(s))$ is the i-th zero of the transfer function $G(s)$

The constrained optimization problem (10), that is usually solved by means of nonlinear programming methods, can be tackled, in this simplified case, by exhaustive search and visual inspection.

Fig. 2 reports the behaviour of the minimum singular value of M as a function of the position h of the strain gauge on a normalized ($L = 1$) beam. In the same figure the sign of the maximum real part of the zeros of $G(s)$ is reported. The sign function takes on value -1 if the zeros of G(s) have all negative real part (admissible region) and $+1$ region if at least one zero of $G(s)$ has a positive real part (inadmissible region). By a simple inspection of figure 2, it ensues that maximization of the minimum singular value of M is obtained when the strain gauge is at the left extreme of the admissible region. By taking into account a normalized tolerance of 5% in placement of the force transducer, the design choice corresponds to fix the strain gauge at position $h^* = 0.5$, i.e., the middle of the beam.

Fig. 2 has been obtained by considering the modal approximation of order 3 ($N = 3$). It can be shown that the behaviour of the minimum singular value of

M does not change if the number of modes included in the approximation increases. However, positions of the transfer function zeros do change with the number of modes. Fig. 3 reports the minimum singular value and the sign of the zeros real part when the order of the modal approximation is four ($N = 4$). It can be seen that the admissible region for a stable inverse has been shrunken, and in particular the position $h^* = 0.5$ now belongs to the inadmissible region.

This means that the optimal design of the sensor needs the *a priori* specification of the maximum number of modes that one wishes to invert for. Such a number may be determined by the dynamic range of the strain gauge, by the precision one wish to achieve and so forth. However, once the choice has been made, and the strain gauge has been placed appropriately, the sensor cannot be used to invert data by using a model with more modes than those previously specified.

Notice also that, as the number of modes increases, the admissible region shrinks progressively towards the free end point of the beam. In the limit case ($N \to \infty$), the only admissible point is the free end point itself, but unfortunately there the system is not invertible, as seen from the minimum singular value, that approaches zero.

4 Inversion Algorithm

In order to determine the applied force, the regularized backward Euler algorithm proposed in (Caiti and Cannata, 1995) has been selected. This algorithm, originated from the numerical study of implicit (or singular) systems, has intrinsic robustness properties, and allows to estimate the input to a system with one-step delay with respect to the measured output.

From dynamic system (5), the following implicit system is built:

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ u \end{bmatrix} + \begin{bmatrix} 0 \\ -I \end{bmatrix} y. \tag{11}$$

The implicit system (11) can be written in the compact form $E\dot{\mathbf{w}} = F\mathbf{w} + Gy$, with obvious meanings. Notice that system (11) is obtained just rewriting the equations of system (5) as an implicit system. By exploiting the non-directionality of implicit representations, it is possible to exchange the role of input and output. Moreover, the system (11) is solvable, i.e., admits unique solution, if and only if the system (5) is invertible (Lewis *et al.*, 1987).

According to (Caiti and Cannata, 1995), the discretized version of system (11) is obtained as

$$\begin{aligned} &(E - \tau(\tfrac{1}{2} + \alpha)F)\mathbf{w}(j+1) = \\ &(E + \tau(\tfrac{1}{2} - \alpha)F)\mathbf{w}(j) + \\ &+\tau G[(\tfrac{1}{2} + \alpha)(y(j+1) - y(j)) + y(j)] \end{aligned} \tag{12}$$

where τ is the sampling period, and $\alpha \geq 1/2$ is referred to as the regularization parameter. For $\alpha = 1/2$ the method above turns into the Backward Euler method. As α increases, the solutions of the discretized equation (12) are low-pass filtered versions of the exact solution, with cut-off frequency progressively decreasing. Furthermore notice that parameter α can be adaptively changed at each computational step.

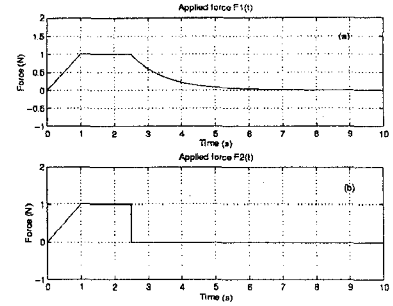

Figure 4: Force $F_1(t)$ end $F_2(t)$ for the simulative test.

5 Simulations

The following simulative cases have been analyzed. A beam of unitary length and homogeneous unitary mass has been considered. Two time-varying forces $F_1(t)$ and $F_2(t)$ in fig. 4 have been selected, one continuous with discontinuous derivative, the other one discontinuous. The strain gauge has been considered as placed at $h = 0.5$ The strain gauge measurements (fig. 5) have been generated in both cases using a truncated model of the 4-th order ($N = 4$), while the estimation has been obtained by using in the inverse system a 3-rd order model. This has been purposefully done in order to investigate the effect of model mismatch disturbances.

The results of the application of the inversion algorithm with $\alpha = 1/2$ in both cases are shown in fig. 6. This figure has to be compared directly with fig. 4. It can be seen that in both cases the applied forces are well reconstructed, notwithstanding the model mismatch problem, *except in the case of jump discontinuity of the applied force.* In the case of a jump discontinuity, the computed inverse solution shows, in correspondence of the jump instant, an impulse-like

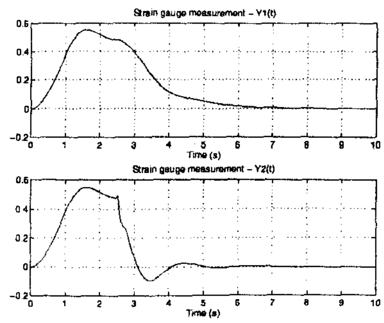

Figure 5: (a): strain gauge measurement $y_1(t)$ obtained for force $F_1(t)$ using a 4 modes approximation. (b): strain gauge measurement $y_2(t)$ obtained for force $F_2(t)$ using 4 modes.

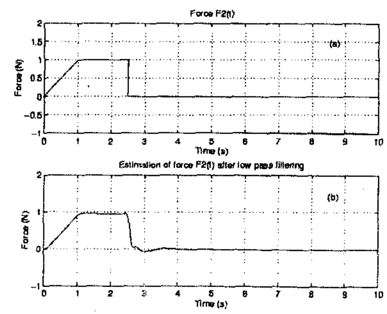

Figure 7: (a) Desired solution ($F_2(t)$). (b) Low-pass filtered signal obtained from signal (b), fig. 6.

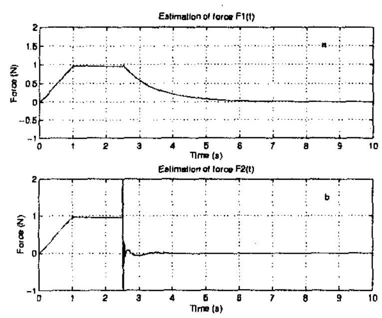

Figure 6: (a) Reconstructed signal using a 3-rd order approximation in building the inverse model, and the backward Euler algorithm for computing the solution. The input to the algorithm is the signal (a) of fig. 5. The desired solution is the signal (a) of fig. 4. (b) Reconstructed signal using a 3-rd order approximation in building the inverse model, and the backward Euler algorithm for computing the solution. The input to the algorithm is the signal (b) of fig. 5. The desired solution is the signal (b) of fig. 4.

spurious response. However, in this case, by simply low-pass filtering the inverse solution (for instance, increasing the value of the parameter α in the inversion algorithm, the signal reported in fig. 7 is obtained. It can be seen that low pass filtering allows for a faithful reconstruction of the input force even in the case of jump discontinuity

6 Experimental Results

The above simulative results have been presented to show that the proposed approach to dynamic force/torque sensing is not only feasible, but can also overcome the obvious problem of *model mismatch* due to the modal truncation. A simple experiment is now reported in order to validate the approach used for the optimal dynamic sensor design.

A steel beam of 0.2m length rigidly fixed at one end is solicited with impulsive forces at the free end. The impulsive forces are given by an operator with a hammer equipped with a piezoelectric sensor able to measure the input signal to the system. A strain gauge has been placed at the position $h = 0.1$m. This is the optimal position for the 3-modes approximation of the system. Direct measurements and a least mean square procedure has been used to identify the physical parameters of the steel beam. Both the hammer input signal and the strain gauge output signal have been acquired with a 12-bit A/D converter with a sampling rate of 2 KHz.

In fig. 8 the actual input signal is described. The normalized strain gauge measurement for this input signal is reported in fig. 9a. Notice that for such signals the transient information is paramount and quasi-static relationships are not useful. Fig. 9b describes the prediction of the strain gauge output obtained with the 3-modes approximation and the input signal in fig. 8. A simple inspection of signals in fig. 9 shows that physical parameters of the system have been identified with satisfactory approximation, and also that three modes are enough to describe the system behaviour. The input estimation obtained by the inversion algorithm is shown in fig. 10. The estimated input must be directly compared with the real input reported in fig. 8. It follows that the input estimate is in excellent agreement with the measured input signal, showing good noise rejection properties and sufficient insensitivity to model mismatch.

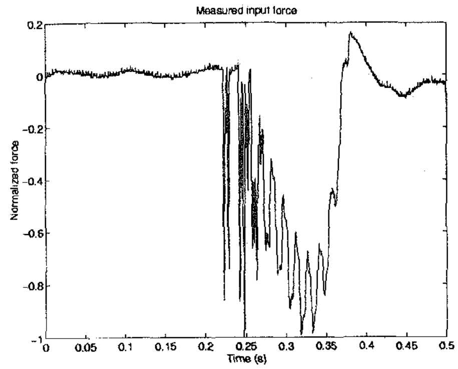

Figure 8: Normalized input force measured from the piezoelectric sensor on the hammer

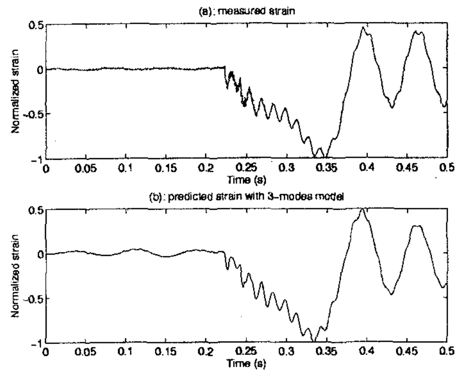

Figure 9: (a-top): normalized strain gauge measurement in correspondance of the input signal of fig. 8; (b-bottom): predicted normalized strain gauge measurement obtained from the 3 modes model of the beam giving as input the signal of fig. 8

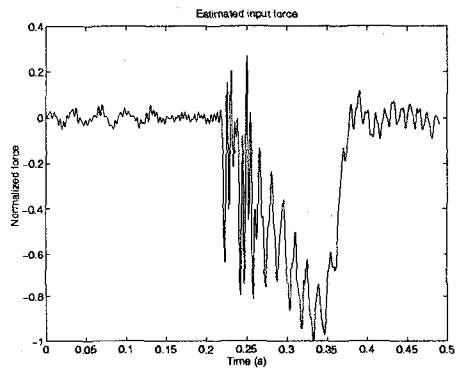

Figure 10: Estimated force input signal with the inversion algorithm based on the 3 modes truncated approximation.

7 Conclusions and Future Work

In this paper an investigation of the optimal design of dynamic force-torque sensors was pursued. The case of a single flexible beam was considered and experimental results were presented. Experimental results show that it is possible to obtain robust solutions to the inversion problem, notwithstanding the use of an approximated model in building of the inverse system. Future developments of this work are foreseen on further experimental investigation of the estimation algorithm, and on the extension of the approach to the case of multiple force/torques acting on the system.

References

Bicchi, A., A. Caiti and D. Prattichizzo (1997). Optimal design of dynamic force/torque sensors. In: *Proc. IFAC Conf. on Control of Industrial Systems.* Belfort France.

Bicchi, A. and G. Canepa (1994). Optimal design of multivariate sensors. *Meas. Sci. Technol.* **5**, 319–332.

Brockett, R.D. and M.D. Mesarovic (1965). The reproducibility of multivariable systems. *J. Math. Analys. Appl.* **11**, 548–563.

Caiti, A. and G. Cannata (1995). Stabilization of spectral methods for the analysis of singular systems using piecewise constant basis functions. *Circ. Sys. Sig. Proc.* **14**, 299–316.

Lewis, F.L., M.A. Christodoulou and B. Mertzios (1987). System inversion using orthogonal functions. *Circ. Sys. Sig. Proc.* **8**, 347–362.

Meirovitch, L. (1967). *Analytical methods in vibration.* Macmillan Publishing Co., Inc.. New York.

Moylan, P.J. (1977). Stable inversion of linear systems. *IEEE Trans. Autom. Contr.* **22**, 74–78.

Sain, M.K. and J.L. Massey (1969). Invertibility of linear time invariant dynamical systems. *IEEE Trans. Autom. Contr.* **14**, 141–149.

Silverman, L.M. (1969). Inversion of multivariable linear systems. *IEEE Trans. Autom. Contr.* **14**, 270–276.

Tan, S. and J. Vandewalle (1988). Inversion of singular systems. *IEEE Trans. Circuit. Sys.* **35**, 583–587.