Mixed Integer Programming for Aircraft Conflict Resolution

L. Pallottino * E. Feron † A. Bicchi ‡

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Abstract

This paper considers the problem of solving conflicts arising among several aircraft. Considering the case when only aircraft heading changes are allowed, we propose a formulation of the multi-aircraft conflict avoidance problem as a mixed-integer linear program, whose solution may be obtained within seconds with standard optimization software. While such a problem formulation and solution is still unsuitable for operational implementation, it may be used as part of a real or fast-time simulation.

1 Introduction

The increasing demand for air transportation is progressively bringing the entire system in an overloaded and congested state. However, the continuing improvement of aircraft instrumentation and communication systems carries the potential of resolving these problems via new air traffic control such as the free-flight concept of operations.

The current enroute air traffic control system consists for the most part of a geographical network in which aircraft are allowed to fly only along fixed routes. The safety of this architecture is supported by many decades of operations. Under this architecture, the dynamics of the air transportation system is dominated by its network structure. Relatively recently, airlines and the Federal Aviation Administration (FAA) have proposed the concept of "Free Flight" ([1], [2]) as a concept of operations relying upon improved communication, navigation and surveillance technology to increase pilot and airline freedom: For example, each pilot would be able to optimize its own trajectory, to minimize the time of flight or to avoid zone of severe weather.

However, the impact of Free Flight upon system safety, as well as the relation between unstructured aircraft flows in Free Flight and air traffic flow management constraints remains largely unknown. Part of building some understanding about Free Flight's safety and efficiency requires building fast simulation environments incorporating automated and optimal conflict detection and resolution schemes. Many approaches have been proposed in the last few years to address the conflict resolution problem when many aircraft are involved; a complete overview of these approaches with a complete bibliography may be found in [5].

The approach proposed in this paper involves centralized, numerical optimization, and is in this regard closely connected to recent approaches proposed by Niedringhaus [6], Durand and Alliot [8] and more recently by Frazzoli *et al.* [7].

We consider the problem of resolving conflicts arising among many aircraft in a cooperative approach, other cooperative approach in ATC literature have been considered [3], [4].

We make the following central assumptions:

- Aircraft are assumed to cruise within a fixed altitude layer. Aircraft can thus be modeled in a purely kinematic fashion, as points in a plane with an associated fore axis, that indicates the direction of motion, and conflict envelope radius. The task of each vehicle is to reach a given goal configuration from a given start configuration (start and goal configurations may represent way points planned for the aircraft by the higher level planner).

*Interdepartmental Research Center "E. Piaggio", University of Pisa

†AIAA Senior Member. Room 35-417, Laboratory for Information and Decision Systems, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA-02139. feron@mit.edu

‡Interdepartmental Research Center "E. Piaggio", University of Pisa

- All interacting aircraft cooperate towards optimization of a common goal, as agents in the same team. This will apply to all aircraft in the same airspace, defined by zone in which they can exchange information on position velocity and goals.

- Aircraft maneuvers consist of instantaneous heading change and then the total path result piece wise linear. If the relative initial distances between aircraft are big "enough" then it is a good approximation of the non linear model described in [4]. In this scenario, the problem of finding the shortest conflict-free paths can be modeled as a Mixed Integer Programming (MIP) problem, which may be solved using optimization tools such as CPLEX [10].

This paper is organized as follows: In the first section we describe the problem and hypotheses to formulate it as a MIP. In the second and third sections we obtain conflict avoidance constraints and formulate them as mixed integer linear constraints. The full mixed integer programming optimization problem is provided in Section 5.

Section 6 presents an extension of the previous problem by allowing aircraft to recover their original trajectory following the conflict avoidance maneuver. In section 7 initial performance estimates are provided comparing initial trajectories with conflict-free trajectories using the proposed optimization framework. Furthermore numerical examples are introduced and solved using CPLEX and performance of the CPLEX resolution for different numbers of aircraft are presented.

2 Problem Statement

In this paper we consider a finite number n aircraft sharing the same airspace; each aircraft has an initial and a final, desired configuration (position, heading) and the same goal: Reach the final configuration in minimum time while avoiding conflicts with other aircraft. A conflict between two aircraft occurs if the aircraft are closer than a given distance d (current enroute air traffic control rules often consider this distance to be 5 nautical miles) [9].

Each aircraft is identified by a point in the plane (position) and an angle (heading angle, direction, that is constant and can be changed with a maneuver) and thus by a point $(x, y, \theta) \in \mathbf{R} \times \mathbf{R} \times \mathbf{S}^1$. Let $(x_i(t), y_i(t), \theta_i(t))$ be the configuration of the i-th aircraft at time t; a conflict occurs when the distance between two aircraft is less than d, i.e. a conflict between aircraft i and j occurs if for some value of t,

$$\sqrt{(x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2} < d. \tag{1}$$

In this paper we restrict to the hypothesis that the aircraft are moving on a horizontal plane and have the same constant normalized linear velocity $v = 1$. The latter assumption may be relaxed at the expense of more notation.

Each aircraft is allowed to make a maneuver at time $t = 0$ to avoid conflicts with other aircraft. The maneuver used in this paper is an instantaneous heading angle change. Finally we assume that no conflict occurs at time $t = 0$.

The problem is to find a minimum heading angle deviation p_i for each aircraft in order to avoid any conflict. In the following sections we formulate conflict avoidance constraints that are linear in those angular deviations p_i.

3 Conflict avoidance constraints

Each aircraft can maneuver only once with an instantaneous heading angle deviation and then we suppose that the i-th aircraft changes its heading angle of a quantity p_i that can be positive (left turn), negative (right turn) or null (no deviation).

The problem then is to find an admissible value of p_i for each aircraft such that all collisions are avoided, the new heading angle and direction of flight is then $\theta_i + p_i$. In this section we formulate some constraints that are linear in the unknowns p_i, $\forall i = 1, ..., n$.

A conflict can occur between pairs of aircraft and then we restrict to the case of two aircraft to obtain collision conditions. Consider two aircraft denoted 1 and 2, respectively.

Let $(x_i, y_i, \theta_i + p_i)$, $i = 1, 2$ be the aircraft's states after the maneuver of amplitude p_i. In this section we show that it is possible to predict the existence of conflicts between the two aircraft based on those aircraft's initial configurations.

Obviously, if the future trajectory for each aircraft (described by a half line) do not intersect, then no conflict can occur. Otherwise a conflict may occur. We now discuss those two cases separately.

3.1 Non-intersecting directions of motion

Consider the case when the geometric half-lines representing the extrapolated trajectories of the two aircraft do not intersect. Consider for example Figure 1: Aircraft 1 has heading angle $\theta_1 + p_1$. Assume the second aircraft is on the straight line forming an angle ω_{12} with the x-axis. If the heading angle $(\theta_2 + p_2)$ of the second aircraft doesn't lie in the outlined sector of amplitude δ then the half lines obtained by projecting forward the motion of both aircraft do not intersect. The condition upon such a case occurs may be expressed easily via some inequality constraints. Let ω_{12} be the angle between the line that joins the aircraft and the x-axis, considering all the possible case of relative position, we have such non-collision constraints that are linear in p_1 and p_2:

$$
\begin{array}{lcl}
\left\{\begin{array}{l}
0 \le \omega_{12} \le \pi \\
\text{ and} \\
\omega_{12} - \pi - \theta_1 \le p_1 \le \omega_{12} - \theta_1 \\
\text{ and} \\
\left\{\begin{array}{l}
-\pi - \theta_2 \le p_2 \le p_1 + \theta_1 - \theta_2 \\
\text{ or} \\
-\pi - \theta_2 \le p_2 \le \omega_{12} - \pi - \theta_2;
\end{array}\right.
\end{array}\right.
& &
\left\{\begin{array}{l}
-\pi \le \omega_{12} \le 0 \\
\text{ and} \\
\omega_{12} - \theta_1 \le p_1 \le \omega_{12} + \pi - \theta_1 \\
\text{ and} \\
\left\{\begin{array}{l}
-\pi - \theta_2 \le p_2 \le p_1 + \theta_1 - \theta_2 \\
\text{ or} \\
\omega_{12} + \pi - \theta_2 \le p_2 \le \pi - \theta_2;
\end{array}\right.
\end{array}\right.
\\
\text{or} & & \text{or} \\
\left\{\begin{array}{l}
0 \le \omega_{12} \le \pi \\
\text{ and} \\
\omega_{12} - \theta_1 \le p_1 \le \pi - \theta_1 \\
\text{ and} \\
\omega_{12} - \pi - \theta_2 \le p_2 \le p_1 + \theta_1 - \theta_2
\end{array}\right.
& \text{or} &
\left\{\begin{array}{l}
-\pi \le \omega_{12} \le 0 \\
\text{ and} \\
-\pi - \theta_1 \le p_1 \le \omega_{12} - \theta_1 \\
\text{ and} \\
p_1 + \theta_1 - \theta_2 \le p_2 \le \pi + \omega_{12} - \theta_2
\end{array}\right.
\\
\text{or} & & \text{or} \\
\left\{\begin{array}{l}
0 \le \omega_{12} \le \pi \\
\text{ and} \\
-\pi - \theta_1 \le p_1 \le \omega_{12} - \pi - \theta_1 \\
\text{ and} \\
\left\{\begin{array}{l}
\omega_{12} - \pi - \theta_2 \le p_2 \le \pi - \theta_2; \\
\text{ or} \\
-\pi - \theta_2 \le p_2 \le p_1 + \theta_1 - \theta_2;
\end{array}\right.
\end{array}\right.
& &
\left\{\begin{array}{l}
-\pi \le \omega_{12} \le 0 \\
\text{ and} \\
\omega_{12} + \pi - \theta_1 \le p_1 \le \pi - \theta_1 \\
\text{ and} \\
\left\{\begin{array}{l}
-\pi - \theta_2 \le p_2 \le \omega_{12} + \pi - \theta_2; \\
\text{ or} \\
p_1 + \theta_1 - \theta_2 \le p_2 \le \pi - \theta_2;
\end{array}\right.
\end{array}\right.
\end{array}
\tag{2}
$$

In the general case of n aircraft we have a group of such constraints for each pair of aircraft (i, j), $\forall i < j$.

3.2 Intersecting directions of motion

For the other cases not included in the previous section we can refer to figure 2, and consider two aircraft (x_1, y_1) and (x_2, y_2) with heading angles θ_1 and θ_2 respectively, we will consider for the moment $p_1 = p_2 = 0$ for simplicity, the general equation will be expressed in the next section. We compute the amplitude $(\theta_1 - \theta_2)$ of the angle formed by the intersection of the aircraft flight directions and the amplitude $(\frac{\theta_1+\theta_2}{2})$ of the angle of his bisector (straight line r) with the x-axis . The bisector r is then a straight line of slope $\tan\left(\frac{\theta_1+\theta_2}{2}\right)$ and the orthogonal to the bisector that passes through aircraft 1 has equation $y - y_1 + (x - x_1)\frac{1}{\tan\left(\frac{\theta_1+\theta_2}{2}\right)} = 0$. The two straight lines that are tangent to aircraft 1 are obtained by the previously defined straight line with a vertical translation of a quantity $\pm v$. It is easy to see that $v = \frac{d}{2\sin\left(\frac{\theta_1+\theta_2}{2}\right)}$.

These two parallel straight lines localize a segment on the straight line followed by the other aircraft. If the disc of radius $d/2$ centered around aircraft 2 intersects this segment then there will be a conflict between the aircraft. This happens when $\left| y_2 - y_1 + (x_2 - x_1)\frac{1}{\tan\left(\frac{\theta_1+\theta_2}{2}\right)} \right| < \frac{d}{\sin\left(\frac{\theta_1+\theta_2}{2}\right)}$. Equivalently a conflict occurs if $\left|\sin\left(\frac{\theta_1+\theta_2}{2}\right)(y_2 - y_1) + (x_2 - x_1)\cos\left(\frac{\theta_1+\theta_2}{2}\right)\right| < d$. Denote with A_{12} the distance between the two aircraft and ω_{12} the angle between the line that joins the aircraft and the x-axis. With this notation a conflict between the two aircraft occurs if

$$\left| A_{12} \cos\left(\frac{\theta_1 + \theta_2}{2} - \omega_{12}\right)\right| < d. \tag{3}$$

For example, in figure 3 four aircraft share the same airspace; equation (3) is represented geometrically by the shadow projected by each aircraft projects onto the other aircraft trajectories. The shadowed area projected by the aircraft A on the trajectory of B, intersects the disc of radius $d/2$ centered in B (and vice versa), hence a conflict between A and B will occur. The disc centered in B doesn't intersect the shadowed area projected by the aircraft C and then no conflict between B and C will occur.

4 Linear constraints formulation

Let now consider n aircraft and their initial configurations $(x_i, y_i, \theta_i + p_i)$, $\forall i = 1, ..., n$. We have shown in previous sections that with some geometric considerations it is possible to predict a conflict between aircraft using only information given by initial states of all the n aircraft. While the constraints given by (2) are linear in the heading angle deviation p_i, the constraints obtained in section 3.2 are not expressed in p_i. We now reformulate them as linear constraints in p_i.

Considering the general case of n aircraft and deviations p_i, from equation (3) a conflict between the aircraft i and aircraft j occurs if

$$\left| A_{ij} \cos\left(\frac{\theta_i + p_i + \theta_j + p_j}{2} - \omega_{ij}\right)\right| < d \tag{4}$$

where we have replaced the heading angle θ_i with the new heading angle $\theta_i + p_i$ after the maneuver of amplitude p_i, where $A_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, and $\omega_{ij} = \arctan\left(\frac{y_i - y_j}{x_i - x_j}\right)$, when $\theta_i + p_i + \theta_j + p_j \neq \pm\pi$. If $\theta_i + p_i + \theta_j + p_j = \pm\pi$ equation (4) is replaced by $|y_j - y_i| < d$.

Let now suppose that $\theta_i + p_i \neq \pm\pi + \theta_j + p_j$, and let define $\alpha_{ij} = \frac{\theta_i + \theta_j}{2} - \omega_{ij}$, in order to avoid collision, the following inequality must be satisfied

$$\left|\cos\left(\frac{p_i + p_j}{2} + \alpha_{ij}\right)\right| > \frac{d}{A_{ij}} \tag{5}$$

In order to avoid collisions each pair of aircraft (i, j) with $i < j$ and such that $\frac{p_i + p_j}{2} + \alpha_{ij} \in [-\pi, \pi]$ must satisfy one of the following inequalities:

$$\begin{array}{c} p_i + p_j \leq 2\arccos\left(\frac{d}{A_{ij}}\right) - 2\alpha_{ij}; \\ p_i + p_j \geq -2\arccos\left(\frac{d}{A_{ij}}\right) - 2\alpha_{ij}; \\ \text{or} \\ p_i + p_j \leq 2\pi - 2\alpha_{ij}; \\ p_i + p_j \geq 2\pi - 2\arccos\left(\frac{d}{A_{ij}}\right) - 2\alpha_{ij}; \\ \text{or} \\ p_i + p_j \leq -2\pi + 2\arccos\left(\frac{d}{A_{ij}}\right) - 2\alpha_{ij}; \\ p_i + p_j \geq -2\pi - 2\alpha_{ij}. \end{array} \tag{6}$$

Given initial configurations (x_i, y_i, θ_i) (before the maneuver), let us introduce the following notation for each aircraft pair (i, j):

- $\alpha_{ij} = \frac{\theta_{i0} + \theta_{j0}}{2} - \omega_{ij}$;

- Upper and lower bound for $p_i + p_j$ for the three *or*-constraints in equation (6)
  - $U1_{ij} = 2\arccos\left(\frac{d}{A_{ij}}\right) - 2\alpha_{ij}$;
  - $L1_{ij} = -2\arccos\left(\frac{d}{A_{ij}}\right) - 2\alpha_{ij}$;
  - $U2_{ij} = 2\pi - 2\alpha_{ij}$;
  - $L2_{ij} = 2\pi - 2\arccos\left(\frac{d}{A_{ij}}\right) - 2\alpha_{ij}$;
  - $U3_{ij} = -2\pi + 2\arccos\left(\frac{d}{A_{ij}}\right) - 2\alpha_{ij}$;
  - $L3_{ij} = -2\pi - 2\alpha_{ij}$;

With those notations, and considering all the possible values of $\frac{p_i+p_j}{2} + \alpha_{ij}$, the constraints (6) may be rewritten as

$$
\begin{array}{l}
\left\{\begin{array}{l}
-2\pi - 2\alpha_{ij} \leq p_i + p_j \leq 2\pi - 2\alpha_{ij} \\
\text{and} \\
\left\{\begin{array}{l}
L1_{ij} \leq p_i + p_j \leq U1_{ij}; \\
\text{or} \\
L2_{ij} \leq p_i + p_j \leq U2_{ij}; \\
\text{or} \\
L3_{ij} \leq p_i + p_j \leq U3_{ij};
\end{array}\right.
\end{array}\right. \\
\text{or} \\
\left\{\begin{array}{l}
p_i + p_j \leq -2\pi - 2\alpha_{ij} \\
\text{and} \\
\left\{\begin{array}{l}
L1_{ij} - 2\pi \leq p_i + p_j \leq U1_{ij} - 2\pi; \\
\text{or} \\
L2_{ij} - 2\pi \leq p_i + p_j \leq U2_{ij} - 2\pi; \\
\text{or} \\
L3_{ij} - 2\pi \leq p_i + p_j \leq U3_{ij} - 2\pi;
\end{array}\right.
\end{array}\right. \\
\text{or} \\
\left\{\begin{array}{l}
2\pi - 2\alpha_{ij} \leq p_i + p_j \\
\text{and} \\
\left\{\begin{array}{l}
L1_{ij} + 2\pi \leq p_i + p_j \leq U1_{ij} + 2\pi; \\
\text{or} \\
L2_{ij} + 2\pi \leq p_i + p_j \leq U2_{ij} + 2\pi; \\
\text{or} \\
L3_{ij} + 2\pi \leq p_i + p_j \leq U3_{ij} + 2\pi;
\end{array}\right.
\end{array}\right.
\end{array}
\tag{7}
$$

5 Problem Formulation

With the obtained constraints the problem cannot be formulated as a mixed integer programming problem because we have many or constraints. We now use a classical approach to formulate or constraints as and constraints.

5.1 Writing *or*-constraints as mixed-integer programming constraints

The conflict avoidance constraints described earlier may be generically written as

$$\begin{cases} c_1 \leq 0 \\ \text{and} \\ c_2 \leq 0 \end{cases} \\ \text{or} \\ \begin{cases} c_3 \leq 0 \\ \text{and} \\ c_4 \leq 0 \\ \text{and} \\ c_5 \leq 0 \end{cases} \\ \text{or} \\ \begin{cases} c_6 \leq 0 \\ \text{and} \\ c_7 \leq 0 \end{cases} \tag{8}$$

where the terms c_i, $i = 1, \ldots, 7$ are linear expressions in the decision variables (heading angle deviations).

The way to transform these *or*-constraint into more convenient *and*-constraint is to introduce Boolean variables. Let t_k with $k = 1, 2, 3$, be a binary number that takes value 1 when one of the or-constraint is active and zero otherwise. For example $f_k = 1$ if constraints c_1 and c_2 are active $f_k = 0$ otherwise. Let G be a large arbitrary number, then the previous set of constraint is equivalent to

$$\begin{array}{l} c_1 - Gf_1 \leq 0 \\ c_2 - Gf_1 \leq 0 \\ c_3 - Gf_2 \leq 0 \\ c_4 - Gf_2 \leq 0 \\ c_5 - Gf_2 \leq 0 \\ c_6 - Gf_3 \leq 0 \\ c_7 - Gf_3 \leq 0 \\ f_1 + f_2 + f_3 \leq 2 \end{array} \tag{9}$$

The last constraint indicates that at least one of the three groups of and-constraints must be verified.

5.2 Variables and constraints

Applying the procedure described in the previous section to the aircraft conflict resolution constraints results in 46 mixed integer linear constraints and 14 Boolean variables for each aircraft pair.

In order to minimize the maximum value of p_i, $\forall i = 1, ..., n$ one might introduce an auxiliary variable q such that $p_i \leq q$, and $-p_i \leq q$ and minimize q. Given n aircraft we have $n(n-1)/2$ aircraft pairs, resulting in a total of n (p_i)+$14n(n-1)/2$ (f_i)+1 (q)= $7n^2 - 6n + 1$ variables and $(46+1)n(n-1)/2 + 2n$ constraints.

Given all the constraints, we can build the matrix A_1, A_2 and the vector B such that the constraints are generically described by the system of linear inequalities

$$A_1 p + A_2 T \leq B \tag{10}$$

Where $p = (p_1, ..., p_n)$, T is the vector that takes account of the Boolean variables and A_1 and A_2 are sparse matrices..

Finally the problem can be formulated as a Mixed Integer Programming problem:

$$\begin{array}{l} \min q \\ A_1 p + A_2 T \leq B \\ p_i \leq q \\ -p_i \leq q \\ f \text{ boolean vector} \end{array} \tag{11}$$

This optimization model can be entered as input to optimization tools such as CPLEX and provably optimal solutions may be obtained.

Another possibility is to minimize the norm 1 of vector $p = (p_1, ..., p_n)$, in order to have a linear cost function other n variables must be inroduced. In simulations described in section 7 we have minimized the infinite norm of vector p.

6 Return maneuvers back to goal configurations

After the first maneuver the aircraft have deviated respect to the original direction (θ_i). In this section we briefly consider one possible option for returning aircraft back to their original desired trajectory.

Consider a distance s that localize the goals positions for each aircraft: $(x_i + s\cos(\theta_i), y_i + s\sin(\theta_i))$, for simplicity the distance between start and goal configuration is equal to s for each aircraft. We want the aircraft to join those configurations avoiding all conflicts, more precisesly we want to find the first instant such that the aircraft can maneuver with another heading angle deviation in order to reach the goal positions without any other conflict.

Referring to figure 4, is not restrictive to consider an aircraft in $(0,0)$ with initial heading angle 0 which have maneuvered with a change p respect to the original angle. After a time t the position of the aircraft will be $(t\cos(p), t\sin(p))$ and then to reach the position $(s, 0)$ another angle deviation is required. The maneuver consists of one heading angle change such that the future direction of motion would be $\frac{p}{|p|}\beta$ where $\beta = \left|\arctan\left(\frac{t\sin(p)}{s - t\cos(p)}\right)\right|$. The problem unfortunately is no more linear in t and we find a good approximation of the minimum time t with an algorithm implemented in MATLAB that increase t if collisions occur and decrease t if no collisions occur.

In figure 5 we have two aircraft that have safety distance $d = 0.1units$ and reach their goal with a path longer than the direct one for $0.0037units$ that had length $6units$.

7 Simulations

In those simulations we have considered the standard value of the safety distance $d = 5$nm. In the first group of simulations we have considered aircraft symmetrically distributed on a circle of radius 60nm centered around the origin (this radius is increased to 100nm for the case of 11 aircraft). Each aircraft is initially headed towards the origin and the goal position is the point on the circle that is symmetric respect to the initial position.

In the absence of maneuver all aircraft would conflict at the origin. In figure 6 we show the scenario of the aircraft if no maneuver is done (left), if the aircraft maneuver optimally to avoid the conflict using the mixed integer programming formulation introduced in this paper (center) and the complete simulation in which each aircraft maneuver back to the goal (right).

In the last simulation (figure 7) we consider 10 aircraft in a shared airspace that are not in a symmetric configurations.

In the next table we indicate the computational time (in seconds) of CPLEX to find the optimal solution to the MIP problem ($TIME$) and the difference of the length of the path that is collision free with the original path during which collisions occur (Δ in nautical miles). Let n be the number of aircraft considered in the simulation:

n	TIME (sec)	Δ (nm)
5	0.34	0.25
7	1.18	0.55
10	5.91	0.45
11	10.4	0.79

The computation times are quite low, compared with other methods used to solve similar problems [7]. Thus this conflict solver may be used in a real or fast-time simulation environment.

8 Conclusions

We have formulated the aircraft conflict resolution problem via heading angle deviations as a mixed integer problem and we have solved it for multiple aircraft using standard optimization software. Optimal solutions have been found quickly (a few seconds) on difficult cases such as the one of 11 aircraft that want to cross the same point at the same time.

After the first maneuver, collisions are avoided but the aircraft are no more directed through their original goal configuration. To address this issue we have developed an algorithm that allow the aircraft to maneuver back to the goal configuration as soon as possible while avoiding the creation of new conflicts.

In future works we will extend to the case of aircraft with different constant speed and aircraft that fly in a three-dimensional space. Future work will also address the case of multi-segmented maneuvers.

Another issue is to consider the MIP problem as an approximation of the non linear problem presented in [4], this can be done considering the perturbation in the B vector in (11).

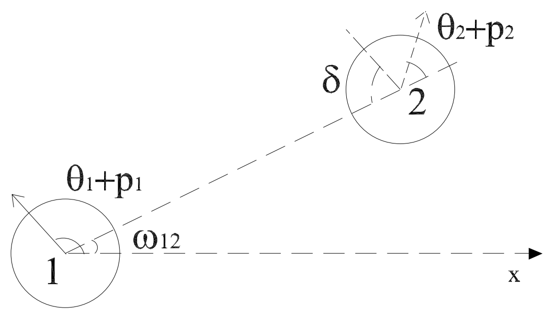

Figure 1: Non-intersecting geometrical trajectories.

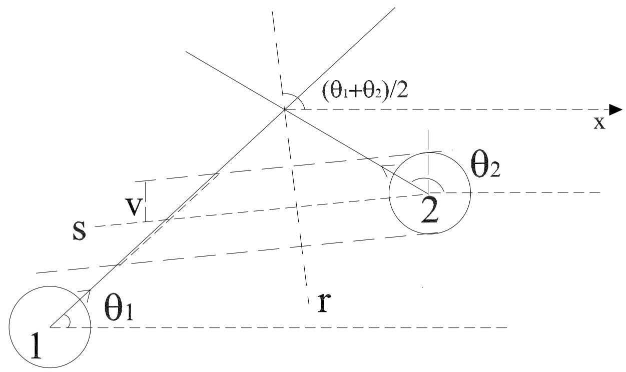

Figure 2: Geometric construction for conflict avoidance constraints in the case of intersecting trajectories.

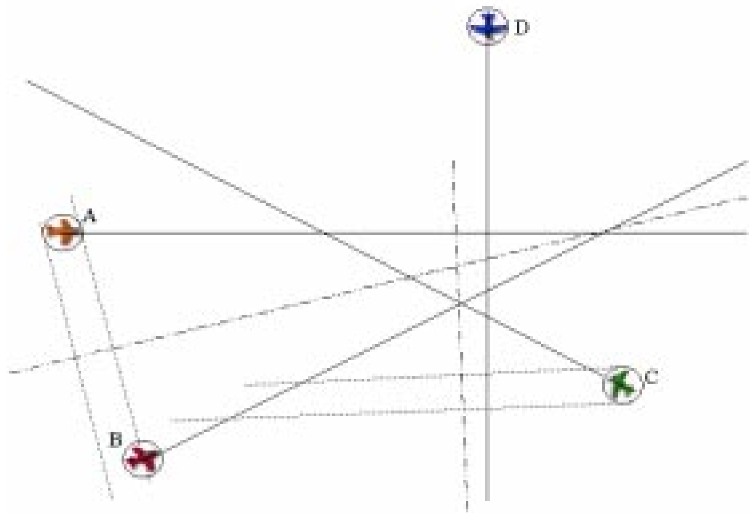

Figure 3: Aircraft flying in a shared airspace, a conflict involving aircraft A and B is predicted because B is in the “shadow” generated by A.

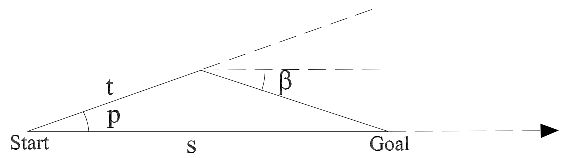

Figure 4: After the maneuver to avoid collision, the aircraft maneuvers to reach the goal configuration

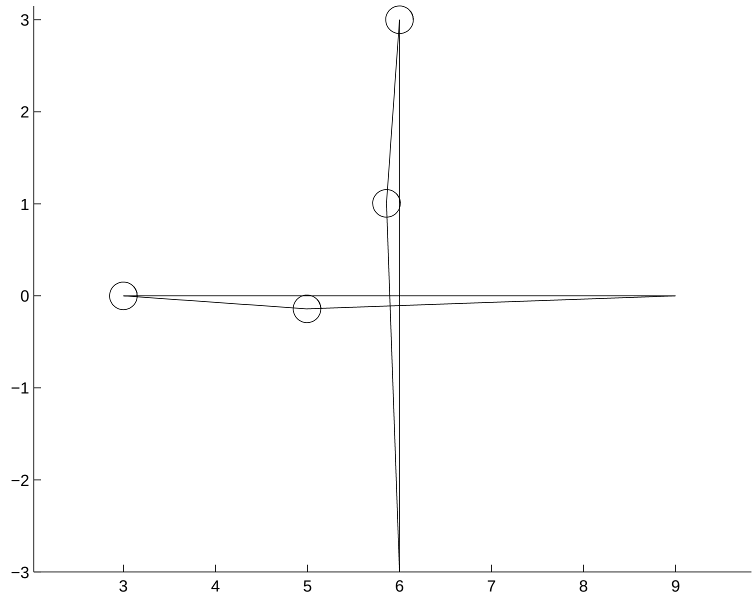

Figure 5: Two aircraft have avoided the conflict and they reach their final goal

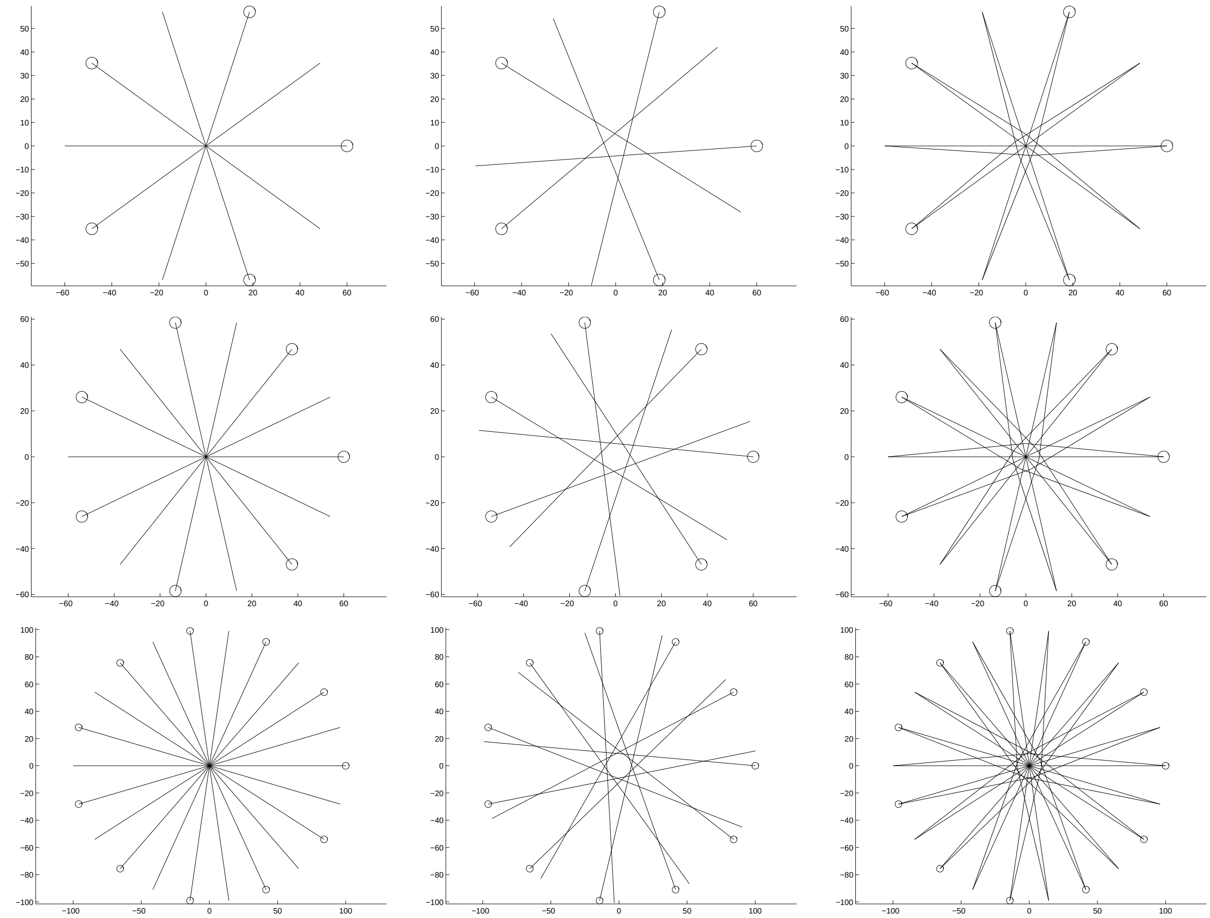

Figure 6: From the top to the bottom we consider the case of 5,7 and 11 aircraft. Left: Scenario with no maneuver. Center: scenario with the non collision maneuver. Right: Complete simulation with two maneuvers.

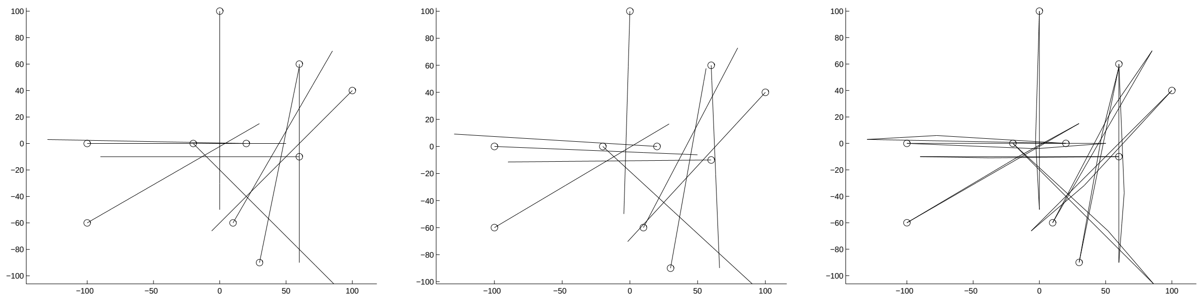

Figure 7: Case of 10 aircraft. Left: Scenario with no maneuver. Center: scenario with the non collision maneuver. Right: Complete simulation with two maneuvers.

References

[1] RTCA Task Force 3, "Final Tech. Rep.: Free Flight Implementation," *Radio Technical Commission for Aeronautics*, October 1995.

[2] Ed. Board, "Special report on free flight," *Aviation Week and Space Technology*, July 1995.

[3] Y.-J. Chiang, J.T. Klosowski, C. Lee, and J.S.B. Mitchell, "Geometric algorithms for conflict detection/resolution in air traffic management,"

[4] A. Bicchi and L. Pallottino, "On Optimal Cooperative Conflict Resolution for Air Traffic Management Systems" to appear in *IEEE Transactions On Intelligent Transportation Systems, Month December, 2000.*

[5] J.K. Kuchar and L.C. Yang, "A Review of Conflict Detection and Resolution Modeling Methods" to appear in *IEEE Transactions On Intelligent Transportation Systems, Month December, 2000.*

[6] W.P. Niedringhaus, "Stream Option Manager (SOM): Automated Integration of Aircraft Separation, Merging, Stream Management, and Other Air Traffic COntrol Functions", *IEEE Transaction on Systems, Man and Cybernetics, Vol. 25, No. 9, September 1995*

[7] E. Frazzoli, Z.H. Mao, J.H. Oh and E. Feron, "Resolution of Conflicts Involving Many Aircraft via Semidefinite Programming" to appear *AIAA, Journal of Guidance, Control and Dynamics, 2001*

[8] Durand, Alliot

[9] H. Erzberger and W. Nedell, *Design of Automated System for management of arrival traffic*, NASA, June 1989.

[10] "Cplex 6.5 Reference Manual", ILOG.