The Drag of Being a Bacterium

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Example of how to write and organize a report for fulfillment of the exam : "Fenomeni di Trasporto Biologico".

Maximum 5 pages. The document should be organised in sections: Abstract, Introduction, Main Text, Conclusion, References. Remember to label graphs and figures and to cite sources. Plagiarism is not acceptable.

Abstract

The viscosity of water is about 0.001 Pa.s, around 1000 times that of air, and its density is also much higher (1000 kg/m^3 compared with 1.2 kg/m^3) making movement far more difficult in aquatic media. Although this may not seem a problem for fish, such as the tuna which can swim at speeds of 20 m/s, for smaller organisms such as bacteria, viscous forces are so large that all events occur in slow motion. This paper describes some of the parameters which make life for bacteria a drag, and explains how flagellar motion drives such organisms through water.

Introduction

Consider an object moving through a fluid. As illustrated in Figure 1, the object encounters three types of resistance. Inertial or pressure drag is due to the amount of fluid the object has to displace as it moves. The second type of resistance is frictional drag, which depends on the surface area in contact with the fluid, and acts parallel to the surface. Finally we have vortex drag which occurs when the fluid moves back into the empty spaces left behind after the object has moved forward, creating a sort of positive pressure gradient.

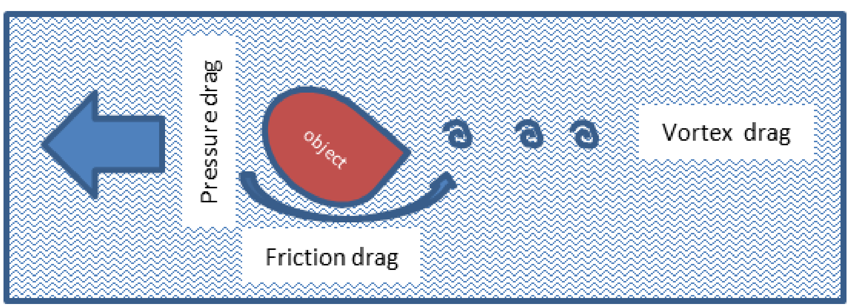

Figure 1: Different types of drag encountered by a moving object

In general the drag of an object with characteristic dimensions L moving at speed U in a medium of density ρ and viscosity μ depends on the Reynolds number (Re). This number is the ratio between inertial and viscous forces per unit volume (Eq. 1).

$$\mathrm{Re} = \frac{\text{inertial forces/unit volume}}{\text{viscous forces/unit volume}} = \frac{\frac{\rho U^2}{L}}{\frac{\mu U}{L^2}} \quad (1)$$

For high Reynolds number flow, the drag forces are predominantly inertial, while for low Re, viscous forces predominate(Eq. 2a and b):

$$\text{inertial drag force} = \rho U^2 L^2 \text{, viscous drag force} = \mu U L \quad \text{(2 a,b)}$$

The drag force (F) for spherical objects is given by the Stokes-Einstein expression, in Equation 3.

$$F = 6\pi L \eta U \qquad (3)$$

Reynolds Number for a bacterium

Bacteria are typically spherical micro-organisms about 1 µm in diameter. Their average speed is about 30 µm/s. In water, the Reynolds number is then 3.10^{-5}, and motion is entirely dominated by viscous forces. Purcell describes some of the consequences of life at low Reynolds number, in which inertia is of no consequence in a well-known and easy to read essay [1]. For example, he calculates how long it would take to a bacterium of radius r=0.5 µm which stopped swimming to come to a complete halt, using the equations (Eqs 5) of motion.

$$\begin{aligned} F &= 6\pi r \mu U = ma \\ a &= \frac{6\pi r \mu U}{m} = \frac{6\pi r \mu U}{\rho \frac{4}{3}\pi r^3} = \frac{9\mu U}{2\rho r^2} \\ t &= \frac{U}{a} = \frac{2\rho r^2}{9\mu} \approx 10^{-7}\,\mathrm{s} \end{aligned} \qquad (5)$$

In these equations m=mass, a =acceleration. The calculation shows that the bacterium basically stops instantaneously. The time of 0.1 µs corresponds to a distance of about 1 picometer, and illustrates that inertia is negligible because the distance is so small compared with the size of the organism. In fact, in this low Reynolds number regime the acceleration terms in the Navier-Stokes equations are insignificant, and thus, as stated in reference 1, "*inertia plays no role whatsoever. If you are at very low Reynolds number, what you are doing at the moment is entirely determined by the forces that are exerted on you at that moment, and by nothing in the past".*

Diffusion in the world of the bacterium

The Peclet number describes the ratio between convection and diffusion times. Let us consider the time it takes for a nutrient molecule to diffuse from a point in space to a bacterium (t_d), and for the bacterium to swim to the molecule (t_s), as illustrated in Figure 2. The ratio of these two times is the Peclet number (Pe), sometimes referred to as the Sherwood number [2].

$$Pe = \frac{t_d}{t_s} = \frac{XU}{D} \qquad (6)$$

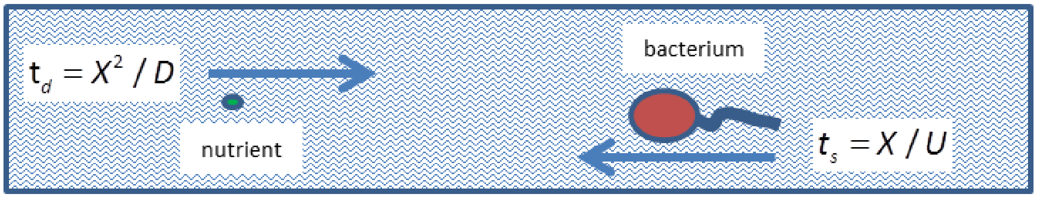

Figure 2: Nutrient diffusion time and characteristic time of a bacterium swimming through water for a travelling distance X.

If Pe=1, the time for the nutrient to diffuse to the bacterium is equal to the time it takes for the bacterium to diffuse to the molecule. If Pe<1, the nutrient will diffuse to the bacterium faster than the bacterium can swim to it. For a molecule with D = 10^{-9} m^2/s (oxygen in water), 1 µm away from the bacterium, Pe=0.03. This low value of Pe tells us that the poor bacterium need not swim at all, but is better off waiting for the nutrient to diffuse towards it.

Why do bacteria swim?

The considerations on Peclet number basically show that *"the fellow* (i.e. bacteria) *who just sits there quietly waiting for stuff to diffuse will collect just as much"* as the one who *"thrashes around" [1]*. So why do bacteria swim? (We know that they do- they are equipped with flagella or little tails which they use to propel themselves). It has been shown that they do not swim to gather food, but rather to move to regions where the food is more abundant (i.e. with a higher concentration). However, to move to a region where the concentration is significantly different, they have to move a distance such that the diffusing molecule is left behind. As Purcell states, they have to *outrun* diffusion. Therefore:

$$\begin{aligned} Pe &= \frac{t_d}{t_s} = \frac{XU}{D} > 1 \\ X &> \frac{D}{U} \end{aligned} \qquad (7)$$

So the bacterium has to move at least 30 µm from its current position in order to find itself in an environment with a different concentration. This will take about 1 second to accomplish.

How do they swim?

When an object pushed backwards on the water, an equal and opposite force is applied by the water on the object: this is known as thrust or push. Keeping in mind the fundamental concept of Re<<1 and the insignificance of inertia, it is clear that the bacterium cannot possibly swim by thrust. In fact, as we have seen, the forces last less than a microsecond and act over distances less than a picometer. To propel themselves, bacteria and other small organisms have developed highly specialized appendages which either rotate (i.e. flagella) or beat continuously from side to side (i.e. cilia), as depicted in Figure 3. Prokaryotes usually have flagella, which rotate like a corkscrew, and this gives rise to their steady, albeit slightly wobbly forward propulsion. One can think of this motion as exploiting frictional viscosity, like the movement of a snake. As the flagella rotates, one component of the motion is directed opposite to the movement of the bacterium, propelling it slowly forwards, while the sideways components cancel each other out.

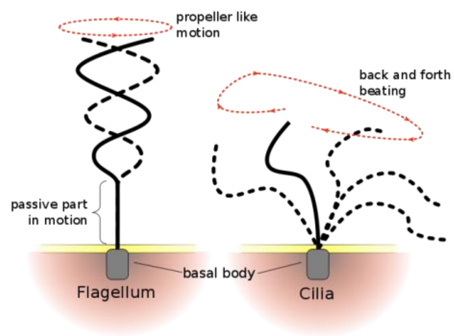

Figure 3: Flagellum and cilia. The former has a rotatory motion while the latter move from side to side (from Wikipedia http://en.wikipedia.org/wiki/Flagellum*).*

Conclusion

This essay gives a brief glimpse into the low Reynolds number world of bacteria and small organisms. I hope you enjoyed reading it. For those interested in further reading, Purcell's paper is an excellent start [1].

References

[1] Purcell, E.M., Life at low Reynolds number. American Journal of Physics Vol. 45, No. 1, Jan. 1977, p. 3-11.

[2]. Vogel S. Life in Moving Fluids. The Physical Biology of Flow, Princeton University Press, NJ. 1996 2nd Ed.